Chapter 5

Exhumation of Ultrahigh-Pressure Rocks: Thermal Boundary Conditions and Cooling History

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Abstract: We investigate the exhumation of ultrahigh pressure (UHP) and high-pressure (HP) rocks in the framework of a dynamic simulation that considers heat advection, heat conduction, heat production, and consequent time-dependent changes in the geothermal gradient. In the absence of lateral heating, rocks exhuming from great depth cool or decompress isothermally and the main cooling period follows the main period of exhumation. Even for a constant exhumation rate, UHP rocks undergo a two-stage cooling history at the end of which the pressure-temperature (P-T) paths of all rocks approach a steady state or "final" geotherm at crustal levels; the shape of the steady-state or final geotherm is mainly a function of exhumation rate. Reconstruction of pressure-temperature-time (P-T-t) paths permits a qualitative distinction between "fast" and "slow" UHP exhumation: fast exhumation is characterized by extremely rapid crustal cooling following small temperature increases or isothermal decompression, whereas slow exhumation is characterized by steady cooling following more modest heating. Rocks exhuming from different depths (e.g., crustal and mantle levels) follow substantially different PT paths (e.g., heating and cooling during decompression), even if all rocks in an orogen are exhumed by the same orogen-scale process.

P-T paths of UHP rocks of the Qinling-Dabie-Hong'an area of central China are consonant with our modeling in that the rocks exhumed from the greatest depth show nearly isothermal decompression, whereas more modestly buried rocks underwent heating during exhumation. The shapes of the P-T-t paths suggest an

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exhumation rate closer to 5 mm/a rather than 1 mm/a, and the apparent two-stage cooling history often interpreted for the Dabie area could have been produced by a single-stage, constant exhumation rate.

1. INTRODUCTION

Deeply exhumed rocks that were metamorphosed under diamond-eclogite or coesite-eclogite facies—ultrahigh-pressure (UHP) rocks—are known from various places around the world (e.g., Liou *et al.*, 1996). Subduction is deemed responsible for the process by which crustal rocks reach depths in excess of 100 km (Hacker and Peacock, 1994), although it remains unclear which boundary conditions allow buoyant continental crust to reach UHP depths (e.g., Molnar and Gray, 1979). Exhumation of UHP rocks from great depth requires either the removal of the overburden by tectonic processes and/or erosion or the transport of the UHP rocks through the overburden (Platt, 1987). This second process is probably important for the emplacement of high-pressure tectonic blocks in melanges of subduction zones (Cloos, 1982) however, we focus on the exhumation of regional UHP terranes. Exhumation via tectonic processes can be either by thrusts and coeval, synthetic normal faults or by conjugate normal faults. In realistic tectonic situations, neither process exhumes rocks unless an additional process such as erosion or special deformation histories like hinge migration or fault rotation are involved.

Although UHP rocks occur in highly deformed zones (e.g., Rubie, 1984; Hacker *et al.*, 1995), structural methods commonly constrain only the final stages of the exhumation process and structures leading to and related to deformation during the UHP event are commonly not observed (Henry *et al.*, 1993). Therefore, most of the information about the UHP event and subsequent cooling has been derived by petrological and thermochronological methods.

Pressure-temperature (P-T) paths of rocks with a high P-T ratio differ from regional metamorphic rocks typical of collision orogens (Ernst, 1988). A pressure maximum at an early stage of orogeny and a subsequent increase in temperature characterize the exhumation histories of most metamorphic rocks from collisional orogens (Thompson and England, 1984). Petrological constraints, however, demand that rocks with a high P-T ratio underwent decompression with little or no heating (Fig. 1, Liou *et al.*, 1996). Some exhumation histories even require a retrograde P-T path subparallel to the prograde path (Cloos, 1982; Ernst, 1988). The establishment of a pressure-temperature-time (P-T-t) path relies on being able to tie P-T conditions to specific times. Thermochronological methods are, in general, restricted to certain closure temperatures, and in particular, the early, hotter stages of the cooling history are often incompletely constrained.

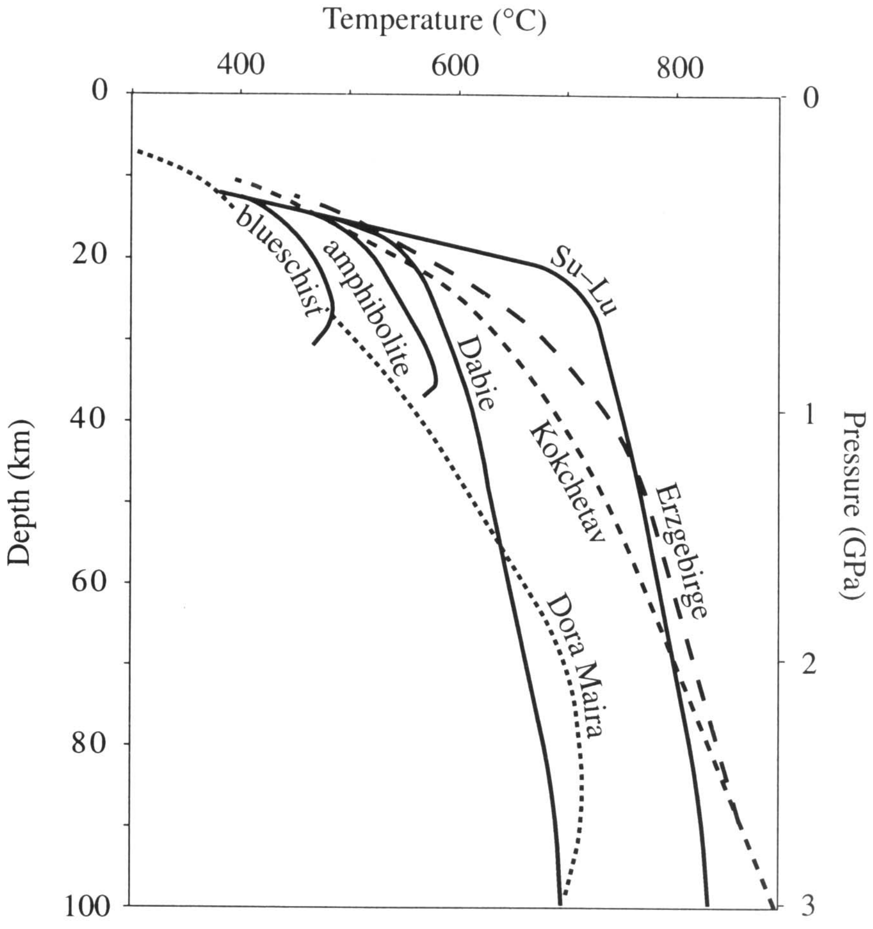

Fig. 1. Decompression paths for UHP rocks from Eurasia and China (Chopin *et al.*, 1991; Schmaedicke *et al.*, 1992; Okay, 1993; Zhang *et al.*, 1994; Liou *et al.*, 1996; Zhang *et al.*, 1997).

The most difficult task is to translate cooling rates into exhumation rates, as this requires needs information about the initial geothermal gradient and the change of the geothermal gradient during exhumation. Here, we investigate exhumation of UHP rocks in the framework of a dynamic simulation that considers heat advection, heat conduction, heat production, and the consequent time-dependent progressive change of the geothermal gradient. The interplay between heat conduction, heat production, heat advection and movement of rocks is complex and dynamic models simulate exhumation more realistically than models with a static thermal gradient (Mancktelow and Grasemann, 1997). An important result of this transient behavior is that heating and cooling of rocks is not only a function of burial and exhumation rates but also of the rate of relaxation of the isotherms. This result underlies why it is invalid to correlate rapid cooling with rapid exhumation.

Based on these physical considerations we aim to clarify by means of one-dimensional thermal calculations and by introducing the instantaneous temperature change (ITC) and the local geothermal slope (LGS) the conditions under which deeply buried rocks heat or cool during exhumation. We show that i) it is unlikely that rocks from great depth heat significantly during exhumation even if exhumation rates are slow, and ii) most of the exhumation occurs well before the rapid cooling recorded by thermochronology. The goal of the presented models is to highlight the influence of some important first-order effects on the exhumation of UHP rocks.

2. TEMPERATURE HISTORY DURING EXHUMATION OF UHP ROCKS

High exhumation rates are typically invoked to explain the preservation of UHP rocks (e.g., Michard *et al.*, 1993; Eide *et al.*, 1994; Nie *et al.*, 1994). However, the temperature history of a metamorphic rock not only depends on the rate of exhumation but also on the initial thermal state of the lithosphere and on the boundary conditions before and during exhumation (Ruppel and Hodges, 1994).

We define the *local geothermal slope,* LGS, as the slope of the temperature gradient at a given depth (Fig. 2):

$$LGS = \lim_{\Delta z_g \to 0} \frac{\Delta T_x}{\Delta z_x} = \lim_{\Delta z_g \to 0} \frac{T_0 - T_1}{z_0 - z_1} \tag{1}$$

where T_g is a temperature range and z_g is a depth interval. If the LGS is positive, temperatures increase downward at that depth; if it is negative, temperatures decrease. Whether a rock cools or heats during movement from depth z_0 to z_1 depends on the transient state of the geotherm. The difference in temperature ΔT over a time increment is defined as positive if the temperature is increasing and negative if the temperature is decreasing. The term *temperature change rate* is preferred to *cooling rate* because the rocks may heat rather than cool during their movement through the lithosphere. The difference in depth Δz is defined to be positive when a rock moves toward the surface.

The rate of change in temperature with respect to change in depth at a given time (ITC; Fig. 2) is defined as:

$$ITC = \lim_{\Delta t \to 0} \frac{\dot{T}}{|\dot{z}|} = \lim_{\Delta t \to 0} \frac{dT / dt}{dz / dt} \tag{2}$$

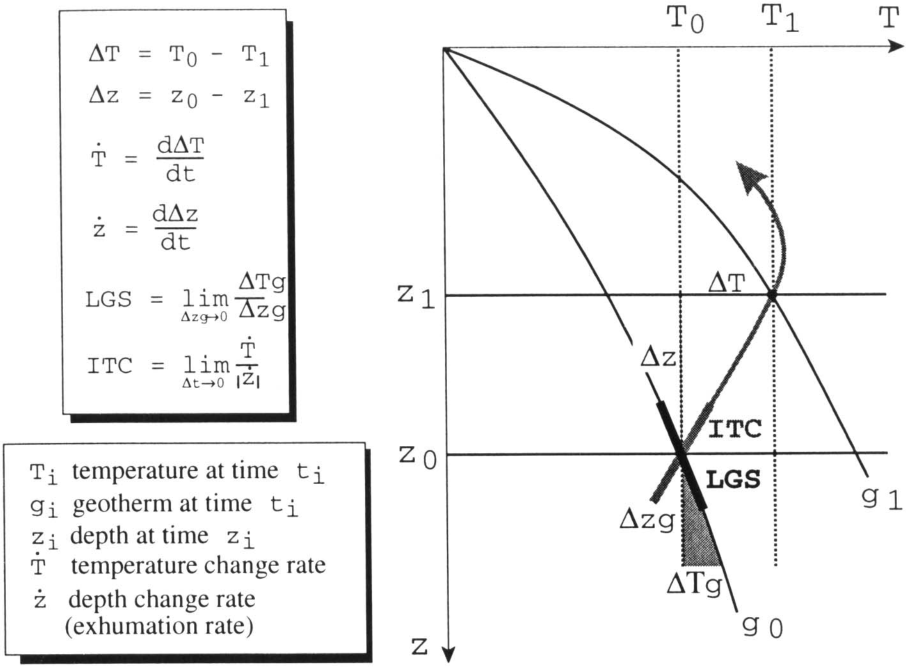

Fig. 2. The instantaneous temperature change (ITC) is the limit of the rate of temperature change divided by the exhumation rate as the time approaches zero. It characterizes the relationship between the change of the thermal regime and the exhumation rate and can be used to indicate if rock at a given depth will heat or cool during exhumation. If the ITC is positive the rocks will heat, if it is negative the rocks will cool. A zero ITC indicates a constant temperature. The relationship between the ITC and the local geothermal slope (LGS) defines whether the rock exhumes above, below, or at steady state.

where $\dot{T}$ is the temperature change rate, $\dot{z}$ is the depth change rate, and Δt is the time increment. By definition, a negative ITC defines heating during exhumation, positive ITC means cooling during exhumation, and an ITC of zero indicates isothermal decompression. When the ITC is equal to the LGS, the rock changes temperature along the steady state geotherm. Equation (2) can also be applied to burial, in which case $\dot{z}$ is negative and represents the burial rate. Therefore, in the general definition of the ITC the term *depth change rate* is preferred. However, the following discussion focuses on exhumation (sensu England and Molnar, 1990), for which $\dot{z}$ is positive. The *steady state geothermal gradient* is the temperature as a function of depth, which does not change within a reasonable limit under the applied boundary conditions. This definition includes quasi-steady state, where mathematically a true steady state cannot be reached (e.g., erosion of a heat-producing layer) but the temperatures do not change significantly over geologic time. For example, at an exhumation rate of 1 mm/a, a long time (>40 m.y.) and corresponding deep exhumation (>40 km) are required to approach steady state (for a more comprehensive discussion of boundary conditions see Mancktelow and Grasemann, 1997). Given that UHP rocks exhume from depth >100 km it is likely that steady state will be reached during exhumation. Considering the possible changes in a geothermal gradient during exhumation the following three regimes are distinguished (Fig. 3), defined by the values of the ITC and the LGS.

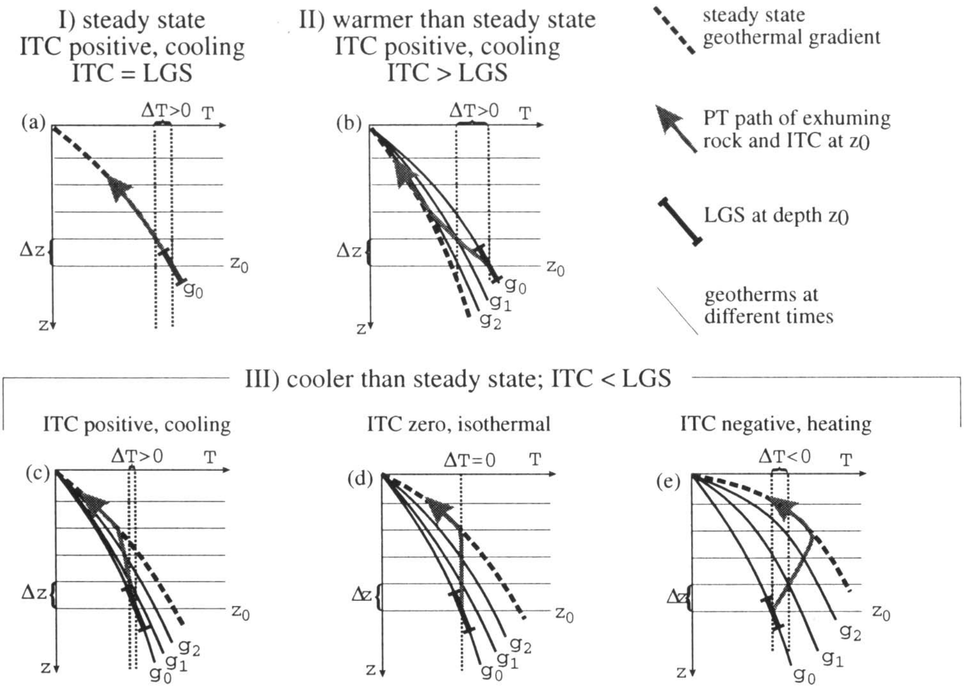

Fig. 3. During exhumation the geothermal gradient can become steeper or gentler. (I) If the ITC and the LGS are equal, rocks exhume at steady state. (II) If the initial geothermal gradient is warmer than the steady state, rocks cool and the ITC is always positive and greater than the LGS. III) If the initial geothermal gradient is cooler than steady state (i.e., ITC < LGS), three different conditions obtain: either the ITC is positive and rocks cool, the ITC is zero and rocks decompress isothermally, or the ITC is negative and rocks cool.

1) **Regime I, ITC = LGS: Steady state exhumation**: The geothermal gradient is at steady state for the given boundary conditions (exhumation rate, radioactive heat production, thickness of lithosphere, etc.). The ITC is equal to the LGS. During exhumation rocks *cool* along the path of the geotherm (Fig. 3a). In special tectonic environments (e.g., long-term subduction refrigeration) the ITC can be locally zero or negative, a situation that is not considered here.

2) **Regime II, ITC > LGS: Warmer than steady state exhumation**: The geothermal gradient is hotter than the steady state condition and the geothermal

gradient relaxes toward lower temperatures. The ITC is always positive and greater than the LGS and rocks at a certain depth must cool during exhumation (Fig. 3b). This situation applies to exhumation after magmatism or where fast exhumation is followed by slower.

3) **Regime III, ITC < LGS: Cooler than steady state exhumation**: The geothermal gradient is cooler than steady state and relaxes during exhumation toward higher temperatures. The ITC is always smaller than the LGS. This is probably the most common situation during the exhumation of rocks because exhumation advects heat to shallow levels, resulting in a steeper steady state geothermal gradient; and crustal thickening or subduction prior to exhumation typically produces an unusually cold geothermal gradient. The latter is considered responsible for the clockwise P-T paths often recorded by amphibolite-facies rocks, where the temperature maximum is reached after the pressure maximum (e.g., England and Richardson, 1977; Spear *et al.*, 1984). The relative rates of advection and conduction determine whether the rocks actually undergo heating or cooling during exhumation:

i) $\Delta T>0$: The ITC is positive and rocks *cool* during exhumation (Fig. 3c).

ii) $\Delta T=0$: The ITC is zero and rocks decompress *isothermally* (Fig 3d).

iii) $\Delta T<0$: The ITC is negative and rocks *heat* during exhumation (Fig 3e).

These regimes are not exclusive and a single exhumation history may be characterized by one or more regimes. Particularly for exhumation from mantle depths, the exhumation history may have two stages. An early stage is characterized by Regime III with continuous relaxation of the geothermal gradient toward steady state where the ITC can have any value. The ITC can even switch between negative and positive during exhumation. A late stage has a stable geothermal gradient for the given boundary conditions and the rocks move toward the surface along the steady state geothermal gradient with a positive ITC (Regime I). A complex change of the exhumation regimes and the ITC values is most likely if the exhumation history is not characterized by a single tectonic event.

The later stages of exhumation, particularly long-term exhumation or exhumation from deep levels, are characterized by thermal steady state (e.g., the paths in Fig. 3, where the arrows follow the dashed steady state curve). In particular, the exhumation of UHP rocks through crustal depths should be along the steady state geotherm. This is only true if the exhumation rate is more or less constant. However, if it is possible to determine this steady state geotherm by means of petrology and/or thermochronology, important parameters like the exhumation rate can be derived (for a more detailed discussion of this idea see below).

An important implication of Equation (2) is that $\dot{z}$ appears as the denominator of a limit function. This implies that even very rapid exhumation cannot change a rock from heating conditions (negative ITC) to cooling conditions (positive ITC) or vice versa. In fact, rapid exhumation rates force the ITC to converge toward zero (isothermal decompression) regardless of whether the initial ITC is positive or negative. The limit of this process is the instantaneous exhumation from z_0 to z_l, resulting in perfectly isothermal decompression with ITC equal to zero. Note that high exhumation rates tend to hold the temperature of an exhuming rock constant. Thus, steady cooling during the whole exhumation period indicates slower exhumation while faster exhumation keeps temperatures relatively constant for a long period of the exhumation history. Thus it is incorrect to equate rapid exhumation with rapid cooling, although rapid exhumation and isothermal decompression are likely to be followed by rapid cooling. This cooling at the end of or after rapid exhumation reflects the continued, slower movement of rocks through closely spaced isotherms or the relaxation of isotherms after exhumation has stopped.

In summary, cooling of rocks during exhumation requires that the transient geothermal gradient be either at steady state (Regime I), warmer than steady state (Regime II), or cooler than steady state (Regime III) with a positive ITC. The latter is most likely to characterize exhumation of UHP metamorphic rocks generated at convergent margins where relatively cold lithosphere is subducted into the mantle. Thus regardless of which exhumation model is favored, exhumation of UHP rocks starts with a geothermal gradient that is cooler than steady state.

3. MODELING OF EXHUMATION

Based on simple boundary conditions we aim to study by means of one-dimensional thermal calculations the first-order effects that influence the exhumation P-T paths of deeply buried rocks. We emphasize that the transient thermal frame of subduction zones in which rocks are brought to great depth can only be described by two-dimensional models (e.g., Minear and Toksöz, 1970; Andrews and Sleep, 1974; Anderson *et al.*, 1978; Hsui and Toksöz, 1979; van den Beukel and Wortel, 1988; Peacock, 1990; Peacock *et al.*, 1994; Ernst and Peacock, 1996). Although the subduction of cool oceanic lithosphere exerts a primary control on the thermal structure of subduction zones by depressing the isotherms, other parameters (e.g., shear heating, induced flow in the mantle wedge, hydrothermal circulations, metamorphic dehydration reaction) influence the transient temperature distribution (Peacock, 1996). The magnitude of these effects is often poorly constrained and we do not consider them in our models.

In the present model we assume that exhumation occurs within a steady state subduction zone. Lateral heat conduction is not considered and therefore the calculated P-T paths are too warm compared with more realistic two-dimensional models where the rising rocks undergo additional heat loss (Ernst and Peacock, 1996). However, the general characteristic of our modeled P-T paths is similar to the more complex models and for high exhumation rates the difference between one- and two-dimensional models is less significant (Ruppel and Hodges, 1994). The advantage of the presented models is that the influence of some important first-order effects can be clearly demonstrated.

To explore exhumation regimes in which the ITC is either positive, zero, or negative, we discuss two models with different boundary conditions at the bottom of the lithosphere. Model I uses a constant temperature-constant depth boundary condition allowing the geothermal gradient to reach steady state. Model II uses a variable temperature-constant depth or constant temperature-variable depth boundary condition, which precludes the establishment of steady state. The commonly used constant heat flux-constant depth or constant heat flux-variable depth boundary condition at the base of the crust (e.g. see discussion in England and Thompson, 1984; Sandiford and Dymoke, 1991) is an inappropriate boundary condition for the base of the lithosphere because unrealistically high steady state temperatures develop at great depth, leading to melting of large parts of the lithosphere. The rheological consequences and the importance of specifying the lower thermal boundary condition are discussed by Stüwe and Sandiford (1995). Temperatures as a function of depth are obtained from the heat transfer differential equation, which is solved numerically by an approach discussed in the Appendix.

3.1 Model I (constant temperature–constant depth)

Model I investigates the thermal evolution of rocks exhumed within the hanging wall of a subduction zone. This tectonic setting has been widely invoked for the exhumation of UHP rocks (e.g., Platt, 1987; Avigad and Garfunkel, 1991; Chopin *et al.*, 1991; Cloos, 1993; Chemenda *et al.*, 1996). The temperature at the surface is 0°C and the lower boundary condition is a constant temperature of 800°C at the base of the hanging wall at 100 km depth. This constant temperature-constant depth boundary condition (Stüwe and Sandiford, 1995) is chosen to model subduction refrigeration. The geothermal gradient chosen for the initial condition is non-steady state and is a low, near-surface gradient of approximately 10°C/km that decreases to 8°C/km at the base of the slab. Other thermal parameters are listed in Table 1.

Results of the model for two different exhumation rates of 1 and 5 mm/a are plotted in temperature-depth plots (Fig. 4). The initial geotherm represents the initial temperature profile before exhumation begins. The final geotherm is the gradient after 100 m.y. for an exhumation rate of 1 mm/a and after 20 m.y. for a rate of 5 mm/a—in both cases, rocks initially buried 100 km are exhumed at the end of the model. The constant temperature at the boundaries (i.e., surface and base of the slab), the long time interval, and the corresponding deep exhumation allow the final temperatures to reach steady state. Mainly due to the advection of heat the steady state geotherms have a pronounced convex-upward shape. The difference in shape between the steady state geotherm of the model with 1 and 5 mm/a is only a result of the different exhumation rates; all other thermal parameters are the same.

Table 1. Thermal modeling parameters.

Parameter	**Model I**	**Model II**
starting depth for exhumation history (km)	10-100	10-100
erosion rate(mm a^{-1})	1, 5	1, 5
temperature of the exposed upper surface (°C)	0	0
depth of lower boundary (km)	100	100 or 140
temperature at lower boundary (°C)	800	600-800°C or 800°C
thermal diffusivity (m^2 s^{-1})	1E-6	1E-6
surface volumetric heat production (W m^{-3})	2.5E-6	2.5E-6
depth at which heat production drops to 1/e (km)	30	30
heat capacity (kJ kg^{-1} K^{-1})	1100.0	1100.0
density (kg m^{-3})	2800	2800

P-T paths for rocks exhuming from 10 to 100 km are shown as loops between the initial and steady state geotherms. The time needed for the exhumation from different depths is different due to the constant exhumation rates. In order to visualize the depths where rocks are heating, cooling, or under isothermal conditions, the diagrams are contoured for negative, zero, and positive ITC. In other words the temperature evolution of a rock for the given boundary conditions at a certain depth and temperature (i.e., the location in the diagrams of Fig. 4) behaves according to the contoured area. Comparing the contoured fields of the ITC in Fig. 4a and b, we stress the following implications:

1) ITC < 0: In a broad range of depths from 5 and 80 km, rocks heat during the initial stages of exhumation. These well-known loop-shaped P-T paths are typical for Barrovian-type metamorphic rocks in collisional orogens (e.g., Thompson and England, 1984). The most extreme ITC values—which correspond to the greatest heating during exhumation—are reached by rocks exhuming from a depth of ~20 km. Higher exhumation rates (Fig. 4b) increase the depth range over which rocks heat, but simultaneously decrease the magnitude of the heating effect. A rock exhuming from 20 km depth at 1 mm/a heats until it reaches a depth of 16 km; the temperature difference is ~40°C. For an exhumation rate of 5 mm/a, heating lasts until the rock reaches a depth of 12.5 km but the temperature difference is only ~20°C.

2) ITC = 0: For reasons of numerical resolution the field of isothermal decompression in Fig. 4 was defined as that part of the exhumation path where temperature changes <2°C. A black bar indicates these quasi-isothermal conditions. This temperature interval is an order of magnitude smaller than the resolution of any geological method. There is only a small depth range around

80 km where exhumation begins isothermally, and this is an effect of the imposed boundary conditions. The duration or the depth range through which rocks exhume isothermally is increased by more rapid exhumation rates: at very high exhumation rates the depth range where rocks decompress isothermally expands toward shallower and deeper levels, consuming the fields of ITC < 0 and > 0. This process is limited by the magnitude of possible exhumation rates (Craw *et al.*, 1994; Burg *et al.*, 1997; Hovius *et al.*, 1997; Blythe, this volume). Another possibility for increasing the size of this isothermal decompression field, not considered here, is variable exhumation rate (Draper and Bone, 1981; Thompson *et al.*, 1997).

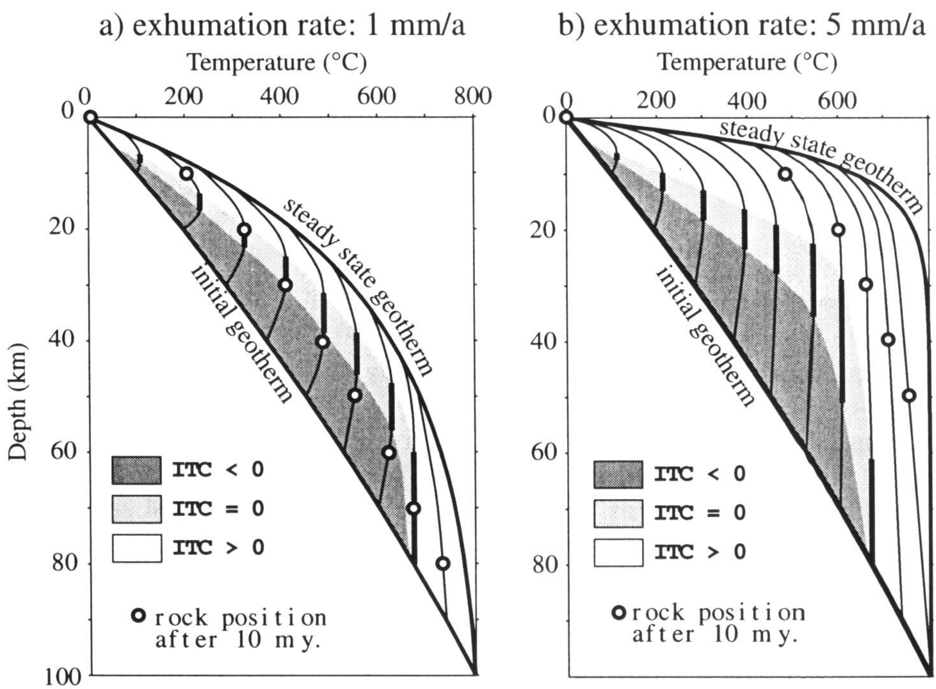

Fig. 4. Results of Model I, where the lower boundary condition at 100 km depth is a constant temperature of 800°C. Temperature/depth plot with an initial geothermal gradient and the steady state condition. P-T-t paths for rocks exhuming from 10-100 km are indicated, and positions of rocks after 10 m.y. of exhumation are indicated with open circles. To visualize where heating, cooling or isothermal conditions prevail, the diagram is contoured for the ITC. Comparison of Fig. 4a and 4b indicates that rocks exhuming from great depths above a constant temperature lower boundary condition always cool during exhumation, regardless of denudation rate. Rocks exhuming from crustal levels heat during their first stage of exhumation.

3) ITC > 0: Any rock that exhumes to the surface must reach the field where the ITC is positive (i.e., cooling). There are two fields where the ITC is greater than 0 at the onset of exhumation. At shallow crustal levels (<5 km) the rocks are

too close to the fixed 0°C surface boundary condition to allow heating during exhumation. At depths >80 km the temperature difference between the initial and the steady state conditions is too small (~100°C) to allow any heating during relaxation. In contrast, the greatest difference in temperature between the initial and steady state thermal gradients occurs at intermediate depths and is ~300°C for the 1 mm/a scenario and 600°C for the 5 mm/a exhumation rate. This is because the chosen initial geotherm is nearly linear (e.g., due to homogeneous crustal thickening) whereas the steady state geotherm has a pronounced convex upward shape produced mainly by advection. Because the heat difference between surface and basal boundary condition within our model lithosphere must remain constant (i.e., 800°C), a steep near-surface thermal gradient (e.g., 60°C/km, Fig. 4b) must be balanced by a gentler gradient at greater depth. This important behavior does not hold true for models with a constant heat flux basal boundary condition at a constant or variable depth (e.g., England and Thompson, 1984), because an increasing geothermal gradient results in increasing temperatures at the base of the slab and a significantly different thermal evolution especially at long time scales (cf. Benfield, 1949; Mancktelow and Grasemann, 1997).

The most important conclusion is that rocks exhuming from mantle depths above a constant-temperature lower boundary condition cool throughout their exhumation regardless of slow or fast exhumation rates. On the other hand rocks exhuming from crustal levels undergo heating followed by cooling. This observation has important implications for the temperature-time histories of exhuming UHP rocks. For example, in an orogen including a suite of rocks exhumed from a range of depths, blueschist-facies rocks should show heating followed by cooling, whereas coesite-containing UHP rocks should show only cooling. Rocks exhumed from depths of 20 km should show the most extreme heating (Fig. 4). Thus, one should question the assumption that all rocks exhumed in an orogen from different depths should follow the same style of P-T path during exhumation.

Another important result of this model is that rocks exhuming under a constant rate from great depth should initially show slow cooling or—within the uncertainty of geological methods—isothermal decompression, followed by much faster cooling. This is the direct consequence of the advection of heat to shallow levels. During the first stage, a rock moves from great depth toward the surface together with the isotherms. However, in the upper part of the crust the rock must pass through narrowly spaced isotherms and cool rapidly. The geologically important implication of this process is that at shallow levels rocks cool along nearly the steady state path (Regime I steady state exhumation). Thus if one can determine the rapidly cooling part of the P-T path of rocks exhuming continuously from great depths, it is possible to approximate the steady state geotherm for this exhumation. As the shape of the steady state geotherm mainly depends on the exhumation rate (compare Fig. 4a and 4b) it is possible, by the thermal modeling techniques outlined here, to estimate the exhumation rate if it is assumed to be constant.

3.2 Model II (variable temperature–constant depth or constant temperature–variable depth boundary condition)

Model II is identical to Model I except that the lower boundary condition is not a constant temperature of 800°C but increases steadily from 600°C to 800°C. Such an increase in temperature is typical for orogenesis where subduction of oceanic lithosphere is followed by subduction of continental lithosphere at slower rates (e.g., Davies and von Blanckenburg, 1995). The mathematical effect of this modified lower boundary condition is that the steadily increasing temperatures at the base of the slab preclude the establishment of steady state during exhumation and the temperatures at all depths (except the surface 0°C boundary condition) increase with time (Fig. 5). Due to the lower temperatures of 600°C at the beginning of exhumation the initial geothermal gradient is slightly less than in Model I.

Comparison of Fig. 4 and 5 shows that an increasing temperature at the base of the slab influences the P-T-t paths of rocks only slightly. For instance, the amount of heating for rocks at depths >90 km is ~10°C. Another result of this lower boundary condition and the fact that steady state can never be established is that rocks, especially those at initial depths >50 km, reach the final geotherm distinctly later than those in Model I. For example, in Model I a rock exhuming from 90 km depth at a rate of 1 mm/a approaches the steady state geotherm at a depth of ~50 km at a temperature >650°C (Fig. 4a). The corresponding rock in Model II approaches the final geotherm at a depth of 30 km at a temperature of only 450°C (Fig. 5a).

The higher exhumation rate dramatically increases the field where the ITC is zero (Fig. 5b). Samples from 60 km down to the base of the slab decompress isothermally for several tens of kilometers. Even at crustal levels where the ITC at the start of the exhumation is negative, rocks heat <10°C. All samples exhuming from 100-140 km depth will steadily cool.

Thus, comparison of Model I and II reveals that whether the boundary condition is constant temperature-constant depth, variable temperature-constant depth or constant temperature-variable depth, rocks exhuming from great depth either decompress isothermally or steadily cool during exhumation. Although there is a temperature difference of as much as 100°C between the steady state geotherms in Fig. 4 and the final geotherms in Fig. 5, the overall shape is very similar and is mainly controlled by the exhumation rate. Even though it might be difficult to estimate these geotherms by petrological methods, a determination of the shape of the P-T path of UHP rocks should allow differentiation between "fast" and "slow" exhumation.

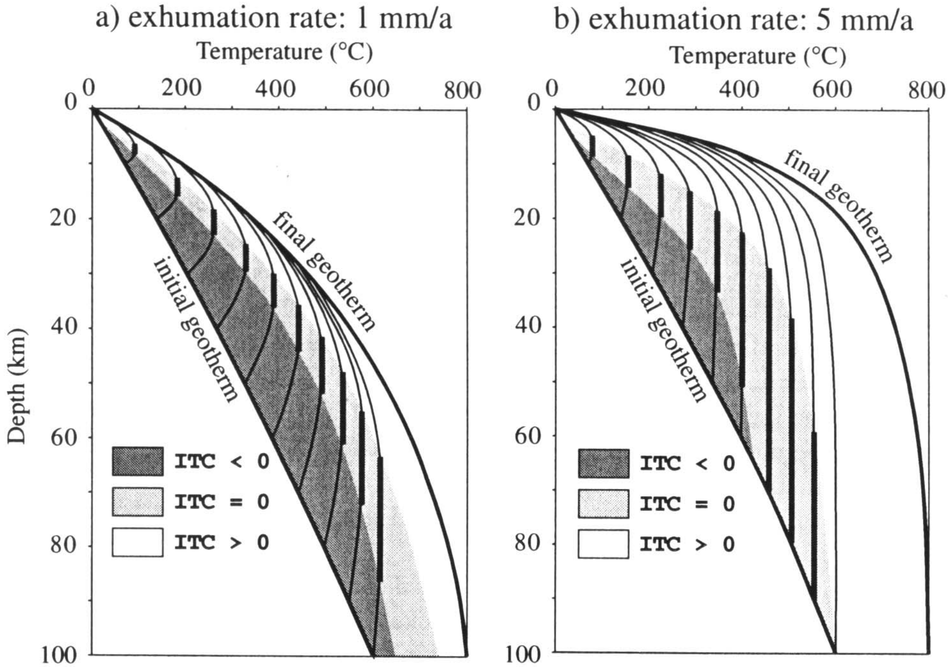

Fig. 5. Results of Model II, where the lower boundary condition at 100 km depth is an increasing temperature from 600 to 800°C. Other parameters are the same as model I. The increasing temperature at the base of the slab precludes that steady state temperature distribution is reached at the end of the exhumation. Comparison of Figs. 4 and 5 shows that an increasing temperature at the base of the slab influences the P-T-t paths of rocks only slightly.

Although the shapes of the P-T paths for rocks exhuming from crustal levels in our models are similar to P-T paths modeling exhumation of a thickened crust with a constant basal heat flux (e.g., England and Thompson, 1984), the boundary conditions are quite different. In our models the constant temperature at the lower boundary condition results in a variable heat flux at the base of the crust, or more precisely, at *any* depth within the exhuming rock column. In fact, the LGS above the lower boundary condition decreases during exhumation until the geotherm reaches a steady state. This has the important consequence that rocks that start to exhume from directly above this constant temperature boundary condition cannot heat during decompression as long as the LGS remains positive. Mancktelow and Grasemann (1997) demonstrated, by comparing analytical solutions for constant heat flux and for constant temperature lower boundary conditions, that the thermal states at crustal levels are similar for both boundary conditions. However, they did not consider exhumation from mantle depths. In the latter case, we have shown that the model results are sensitive to the chosen lower boundary condition.

Draper and Bone (1981) used a constant temperature lower boundary condition to investigate the thermal evolution and preservation of blueschist terranes. Although they came to the similar conclusion that rocks exhuming from lower crustal level experience the maximum temperature rise, they did not consider the exhumation of rocks directly from above the lower boundary condition. Therefore they came to the different conclusion that all rocks exhuming with constant exhumation rates from greater depth heat during decompression.

4. EXHUMATION RATES AND COOLING RATES IN UHP ROCKS

As discussed above, the exhumation of rocks from UHP depths of 100 km is most likely accompanied by cooling or isothermal decompression. The exhumation rate as denominator in equation (2) means that rapid exhumation causes UHP rocks to approach isothermal decompression, while slow exhumation favors a steady cooling. The different cooling histories of rocks exhuming at different rates are illustrated in Fig. 6. In this plot cooling curves for rocks exhuming from 90 km depth by 1, 2, 3 and 5 mm/a are calculated. The time axis in this plot is normalized to facilitate comparison of the different exhumation rates. All other thermal parameters and boundary conditions are the same as in Model I.

As outlined above, the advection of heat widens the spacing of the isotherms at depth, i.e., it lowers the LGS, and the isotherms are more densely spaced against the surface boundary condition, i.e., the LGS is elevated. This effect increases dramatically with higher exhumation rates. Rocks that move through a temperature field where the distance between isotherms is increasing cool slowly or maintain constant temperature if the exhumation rate is equal to the velocity of the isotherms. Rocks exhuming through closely spaced isotherms record fast cooling (Fig. 6). In Fig. 6, only 10% of the total exhumation history of rock *a* is isothermal; the rest of the cooling history shows steadily increasing cooling rate. Thermochronological methods using closure temperatures below 600°C would record only about the last 40%, or 35 m.y., of the cooling history. This is a dramatic difference from rock *d* exhuming at higher rates; for this rock, 75% of the exhumation history is isothermal and thermochronological methods recording cooling below 600°C would only cover the last 10%, or ~2 m.y.

These considerations have important applications for constraining the exhumation rates and periods of UHP rocks. The cooling of UHP rocks detected by thermochronological methods records only a late stage of the whole exhumation history that began much earlier. Although it may be difficult to determine the exact cooling history of UHP rocks, Fig. 6 can be used as a qualitative guide to the duration of exhumation prior to that constrained by geochronometry. It is thus conceivable that most UHP rocks began to exhume at an early stage of an orogenic cycle. This is in good agreement with geological records and exhumation models suggesting syn-convergent exhumation (e.g., Platt, 1987; Avigad and Garfunkel, 1991; Chopin *et al.*, 1991; Cloos, 1993; Chemenda *et al.*, 1996).

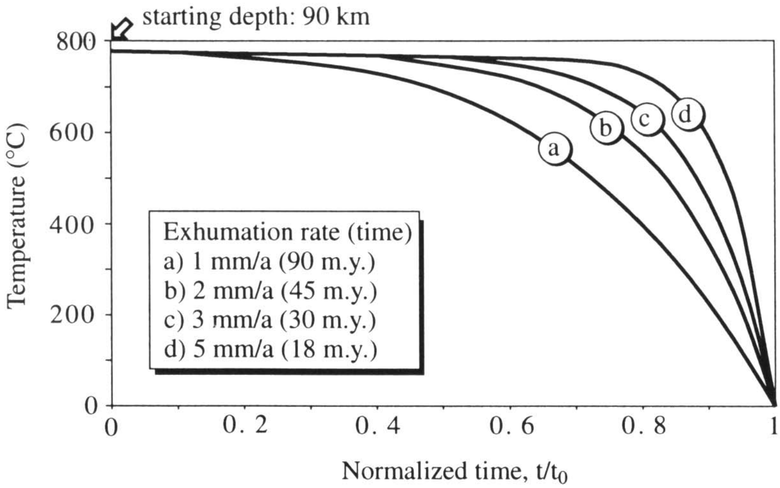

Fig. 6. Cooling curves for rock volumes exhuming from 90 km depth at 1, 2, 3 and 5 mm/a respectively. The time axis is normalized to compare the different time spans needed for the total exhumation. Boundary conditions and thermal parameters the same as Fig. 3. All curves show initially nearly no cooling and cooling rates increase steadily with time. Regardless of the exhumation rate, the exhumation from great depth starts significantly before the onset of cooling.

In summary, for the boundary conditions investigated, UHP rocks can be preserved under a wide range of exhumation rates. By careful examination of the geological data, it should be possible to distinguish between "slow" and "rapid" exhumation of UHP rocks. Slow (~1 mm/a) exhumation (Fig. 4a, 5a and 6) should be characterized by i) a moderate, e.g., ~20-30°C/km near-surface, steady-state geothermal gradient during exhumation through the *crust*; ii) deeply exhumed rocks should show nearly isothermal cooling; iii) shallowly exhumed rocks should exhibit modest temperature increases of 50-100°C, followed by more substantial cooling; and iv) thermochronometers with closure temperatures of 300-600°C should record an age range of 10-20 m.y. In contrast, rapid (~5 mm/a) exhumation (Fig. 4b, 5b and 6) should be characterized by i) an extremely high near-surface geothermal gradient, e.g., ~60-70°C/km at the late stage of exhumation; ii) deeply exhumed rocks should show isothermal cooling to even shallower levels followed by very rapid cooling; iii) shallowly exhumed rocks should exhibit smaller temperature increases of <50°C, again followed by rapid cooling; and iv) thermochronometers with closure temperatures of 300-600°C should record an age range of ~1 m.y.

5. APPLICATION OF THE MODELING TO THE QINLING-DABIE OROGENIC BELT, CENTRAL CHINA

The Su-Lu-Dabie-Hong'an areas contain regional UHP and HP metamorphic rocks developed during a Triassic orogenic event (e.g., Hacker *et al.*, 1995). Current constraints on the P-T-t paths of these rocks are summarized in Fig. 1; the true P-T-t paths were undoubtedly more complicated. Rocks exhumed from the greatest depths, >120 km, exhibit apparent isothermal decompression or cooling, with a marked inflection at around 0.6–0.8 GPa (Liou *et al.*, 1996). High-pressure amphibolite and blueschist-facies rocks reached peak P conditions of ~1.2 GPa, ~550°C and 0.4–0.8 GPa, 300-457°C, respectively, and underwent 25–50°C heating during decompression (Eide, 1993). Among the northernmost UHP rocks in the Dabie Shan, U/Pb zircon dating indicates peak temperatures (700°C) prevailed at ~219 Ma (Ames *et al.*, 1996; Rowley *et al.*, 1997), and $^{40}Ar/^{39}Ar$ phengite and K-feldspar spectra show cooling to temperatures <300°C by ~190 Ma (Hacker and Wang, 1995; Hacker *et al.*, in press).

The Qinling-Dabie-Hong'an P-T paths are consonant with the modeling presented here in that the rocks exhumed from the greatest depth show nearly isothermal decompression, whereas more modestly buried rocks underwent heating during exhumation. If we assume that the exhumation rate was constant, by reference to Fig. 4 and 5, the shapes of the P-T paths of these Chinese rocks suggest an exhumation rate closer to 5 mm/a rather than 1 mm/a. By comparison, assuming that the 219 Ma zircons formed at 3 GPa (Ames *et al.*, 1996) and the 190 Ma ages record cooling to <1 GPa (Hacker and Wang, 1995), yields an exhumation rate of ~2 mm/a.

Strictly, U/Pb zircon ages record the time of zircon crystallization and not necessarily the time of peak pressure. Thus the zircon ages might indicate cooling to temperatures of ~700°C at pressures considerably below maximum (Hacker *et al.*, in press). If so, Fig. 4–6 imply that the main 30 m.y. cooling period followed an equally long period of exhumation from deeper levels.

The P-T paths of Fig. 1 have often been used to argue for a two-stage exhumation history reflecting differential rates of exhumation through mantle and crustal depths (e.g., Liou *et al.*, 1996). Figs. 4–6 reveal, however, that an apparent two-stage cooling history could have been produced by a constant rate of exhumation.

6. CONCLUSIONS

The following points are based on the presented one-dimensional models and thus investigate first-order processes. For quantitative analysis, a two-dimensional approach including additional thermal effects such as induced mantle convection, slab age, fluid production, hydrothermal circulation, and shear heating is required (e.g., Peacock, 1990; Peacock, 1996).

1) The local geothermal slope (LGS) and the rate of change in temperature with respect to change in depth at a given time (ITC) are tools for describing when rocks heat, cool or behave isothermally during exhumation or burial.

2) In the absence of lateral heating, rocks exhuming from great depth always cool or decompress isothermally.

3) The main cooling period follows the main period of exhumation.

4) Shallowly buried rocks follow substantially different P-T paths than deeply exhumed rocks even if all rocks are exhumed by the same orogen-scale process. This observation has important consequences for the interpretation of metamorphic coherent HP terranes with rocks recording different P-T paths.

5) Rocks undergo two-stage cooling histories even for a constant exhumation rate. If the exhumation rate is constant, the P-T paths of all rocks approach a steady state or "final" geotherm at crustal levels, the shape of which is mainly a function of exhumation rate. If it is possible to determine this steady state geotherm by geological methods, exhumation rates can be estimated by reference to the models presented here.

6) After long time scales and corresponding deep exhumation, the geotherms approach a steady state condition at crustal levels. Therefore, reconstruction of P-T-t paths should permit distinction between "fast" and "slow" UHP exhumation processes. For example, fast exhumation is characterized by extremely rapid crustal cooling following small temperature increases or isothermal decompression, whereas slow exhumation is characterized by steady cooling following more modest heating.

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APPENDIX

The temperature distribution in a homogeneous isotropic solid whose thermal material parameters are independent of temperature is a particular solution (for given boundary conditions) of the heat transfer differential equation. In two dimensions with only a vertical velocity component this equation reduces to:

$$\frac{\partial T}{\partial t}=\kappa\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)+\left(\dot{z}\frac{\partial T}{\partial z}\right)+\frac{A(z,t)}{\rho C} \qquad (3)$$

where

T = temperature (°C)

t = time (s)
κ = thermal diffusivity (m^2/s)
x, z = horizontal and vertical (i.e. depth) direction (m)
$\dot{z}$ = exhumation rate (m/s)
$A(z,t)$ = volumetric heat production (W/m^3)
C = specific heat (J/kgK)
ρ = density (kg/m^3)

The equation states that the change of temperature with time depends on heat conduction, advection, and production. This equation was solved numerically over a finite difference grid with a vertical distance of 100 km representing the thickness L of the lithosphere. The equally spaced grid resolution is 100 m. The volumetric heat production was modeled with an exponential function where the heat generation decays with depth:

$$A(z,t) = A_0 \exp\left(\frac{-z}{l(t)}\right) \tag{4}$$

where A_0 is the heat generation in the surface layer, l is the variable depth that changes with time at which the heat production drops to 1/e of this surface value. This system is solved numerically in two dimensions using an ADI (alternating-direction-implicit) method with a two-step scheme (Peaceman and Rachford, 1955). Although the mathematical description and solution used in the calculations is two-dimensional, only the vertical velocity component (i.e., the exhumation rate) was considered and thus reduces strictly speaking to a one-dimensional problem. The advection term was solved separately using an upwind differencing scheme, which applies backward or forward differencing depending on the sign of the velocity vector. This procedure is only stable as long as

$$\Delta t \leq \left\{\frac{|\dot{x}|}{\Delta z^2} + \frac{2x}{(\Delta z)^2}\right\}^{-1} \tag{5}$$

The other boundary conditions are:

$$T = T_s = 0 \tag{6}$$

at the upper boundary where T_s is the constant surface temperature, and

$$\frac{dT}{dX} = 0 \tag{7}$$

at the left and right sides. The lower boundary condition is either a constant temperature T_l for Model I or a linear function $T_l(t)$ for Model II that increases the temperature during the exhumation process. The algorithm was tested and compared with a one-dimensional analytical solution for the time variation of temperature from an initial steady-state geotherm without erosion to the re-establishment of a steady state following onset of erosion at a constant rate (Mancktelow and Grasemann, 1997):

$$T(z,t) = \zeta(z,t)\exp\left(-\frac{\dot{z}}{2\kappa}z - \frac{\dot{z}^2}{4\kappa}t\right) + T(z) \tag{8}$$

where T(z) is the steady state solution:

$$T(z) = \beta\left[1 - \exp\left(-\frac{z}{l}\right)\right] + \gamma\left[1 - \exp\left(-\frac{\dot{z}}{\kappa}z\right)\right] \tag{9}$$

and

$$\beta = \frac{Al^2}{\rho C(\kappa - \dot{z}l)} \tag{10}$$

A solution for the time and position dependence is given by Carslaw and Jaeger (1959):

$$\zeta(Z,T) = \frac{2}{L}\sum_{n=1}^{\infty}\exp\left(-\kappa\omega_n^2 t\right)\sin\omega_n z\int_0^L \zeta(z',t=0)\sin\omega_n z' dz' \tag{11}$$

where

$$\omega_n = \frac{n\pi}{L} \tag{12}$$